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Technical Notes

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Heat Release Effects on the Instability of Parallel Shear Layers

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Nomenclature

a = isentropic speed of sound
B' = Bernoulli enthalpy
C_p = specific heat at constant pressure
h = specific enthalpy
p = pressure
$\dot{Q}$ = heat addition rate per unit volume
s = specific entropy
T = temperature
t = time
$\boldsymbol{u}'$ = rotational component of velocity fluctuations
$\boldsymbol{v}$ = velocity vector
ϕ = acoustic potential
ρ = density
γ = ratio of specific heats

Superscripts
$'$ = time dependent component
$-$ = time average

Introduction

THE influence of time-dependent heat addition on the linear instability of shear layers is of considerable interest in understanding the dynamic behavior of reacting flows and combustion-turbulence interactions. Several recent numerical studies[1–4] have, therefore, been conducted to clarify the characteristics of parallel shear layers, particularly in the low Mach number limit[1–3] where acoustic waves are reportedly "filtered" out. These studies have shown that the heat release, usually due to chemical reaction, affects the instability characteristics primarily due to its influence on the mean (time-averaged) profiles of velocity and temperature. The time-dependent component of the heat release, according to these computations, is not important in determining the instability characteristics. However, the reason why the shear layer instability characteristics are not sensitive to the heat release rate oscillations is not clarified in these reports. It is shown in this Note that by adopting a kinematic definition of acoustic waves it is readily apparent that heat release rate oscillations serve as a source of sound and have no direct effect on the shear layer instability characteristics when acoustic waves are negligible.

Analysis

The approach is based upon the Bernoulli enthalpy aeroacoustics theory,[5–7] which utilizes the specific enthalpy h and specific entropy s as the primary thermodynamic variables. In addition, velocity oscillations $\boldsymbol{v}'$ are spilt into rotational $\boldsymbol{u}'$ and irrotational $\nabla\phi$ components by means of the Helmholtz decomposition theorem.

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The linearized equations for the time-dependent components of the flow variables may be written in the following form:

$$\frac{\bar{D}^2\phi}{Dt^2} - \bar{a}^2\nabla^2\phi - (\nabla\phi\cdot\nabla)\bar{h} + \gamma\bar{T}(\nabla\phi\cdot\nabla)\bar{s} - \frac{1}{C_p}\frac{\bar{D}\phi}{Dt}(\bar{\boldsymbol{v}}\cdot\nabla)\bar{s} = \frac{\bar{D}B'}{Dt} - \gamma\bar{T}\left[\frac{\bar{D}s'}{Dt} + (\boldsymbol{u}'\cdot\nabla)\bar{s}\right] - \frac{1}{C_p}B'(\bar{\boldsymbol{v}}\cdot\nabla)\bar{s} + (\boldsymbol{u}'\cdot\nabla)\bar{n} \tag{1}$$

$$\frac{\bar{D}\boldsymbol{u}'}{Dt} + (\boldsymbol{u}'\cdot\nabla)\bar{\boldsymbol{v}} = -\nabla B' + \bar{T}\,\nabla s' + \frac{B'}{C_p}\nabla\bar{s} - \frac{1}{C_p}\frac{\bar{D}\phi}{Dt}\nabla\bar{s} + \nabla\phi\boldsymbol{x}(\nabla\boldsymbol{x}\bar{\boldsymbol{v}}) \tag{2}$$

$$\frac{\bar{D}s'}{Dt} + (\boldsymbol{u}'\cdot\nabla)\bar{s} = \left(\frac{\dot{Q}}{\rho T}\right)' - (\nabla\phi\cdot\nabla)\bar{s} \tag{3}$$

where B', the Bernoulli enthalpy, is defined by

$$B' = h' + \frac{\bar{D}\phi}{Dt} \tag{4}$$

and $\bar{D}/Dt$ is the substantial derivative following the time-averaged fluid motion

$$\frac{\bar{D}}{Dt} = \frac{\partial}{\partial t} + (\bar{\boldsymbol{v}}\cdot\nabla) \tag{5}$$

Note that Eq. (1) is the acoustic wave equation for ϕ which, therefore, may be identified with the acoustic potential. To eliminate the effects of acoustic waves from the analysis, it therefore suffices to drop the ϕ terms in the preceding equations. Furthermore, when $\nabla\bar{p} = 0$ (appropriate for parallel shear layers) the right-hand side of Eq. (2) is expressible as $-\nabla p'/\bar{\rho}$, where p' is now purely hydrodynamic since the acoustic component has been suppressed. Thus, the instability of the shear layer in the absence of acoustic waves (or when they are negligible) may be studied by means of the momentum equation

$$\bar{\rho}\left[\frac{\bar{D}\boldsymbol{u}'}{Dt} + (\boldsymbol{u}'\cdot\nabla)\bar{\boldsymbol{v}}\right] = -\nabla p' \tag{6}$$

subject to the condition

$$\nabla\cdot\boldsymbol{u}' = 0 \tag{7}$$

which arises from the Helmholtz decomposition of the velocity oscillations.

These equations are independent of the unsteady component of the heat addition rate and are readily shown to be identical for the case $\dot{Q}' = 0$ when acoustic waves are neglected. Hence, by adopting a kinematic definition of sound, it is seen that unsteady heat addition has no direct effect on shear layer instability. The entropy wave s' does depend on Q', but may be computed via Eq. (3) after obtaining $\boldsymbol{u}'$ and p'.

It may be noted that formulations using either thermodynamic or kinematic definitions of sound lead to the same result for the case of a vanishing time-dependent component of the heat addition rate. Using the thermodynamic definition, acoustic waves are suppressed by neglecting the pressure perturbation in the equation of state so that

$$\bar{T}\rho' + \bar{\rho}T' = 0 \tag{8}$$

Then, utilizing the linearized continuity and energy equations (with no time-dependent heat source) results in the condition that the velocity oscillations are divergence free, which recovers Eq. (7). The linearized momentum equation is still given by Eq. (6) so that the correspondence with the suppression of acoustic waves using the kinematic definition of sound is complete for this case.

Acknowledgment

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Long Distance Propagation Model and Its Application to Aircraft En Route Noise Prediction

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Introduction

As a member of the SAE A-21 Aircraft Noise Committee En Route Noise Group, McDonnell Douglas Aerospace (MDA) participated in the Aircraft En Route Noise Technology Program developed by NASA/FAA. As part of the research activities of this program, two en route noise tests were conducted, one in Madison and Limestone Counties, Alabama in 1987, and the other in White Sands Missile Range, New Mexico in 1989. These tests were designed to address the atmospheric propagation of advanced turboprop noise from cruise altitudes.

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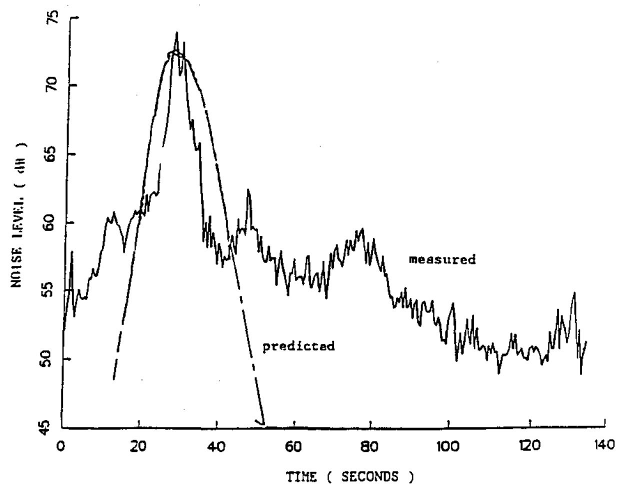

Fig. 1 Comparison of measured and predicted en route noise time histories: run 202, flight altitude = 4.6 km (15,000 ft), BPF = 233 Hz.

During this period, MDA developed a three-dimensional long distance propagation model designed to be used exclusively for en route noise prediction. In this paper the structure of the model is outlined, and its applications to en route noise predictions are also discussed.

Theoretical Background

The long distance propagation model presented here was developed based on the ray theory of Ref. 1. The model is designed to be used exclusively for aircraft en route noise prediction in the stratified atmosphere. Two major assumptions were adopted in this model: 1) The atmosphere-related parameters such as pressure, temperature, wind velocity, and humidity should not change appreciably in the vertical direction over the wavelength of the sound whose propagation is in concern. 2) The vertical component of the wind velocity is negligibly small. Then, the governing equations for the long distance propagation are written as

$$\begin{aligned} x(z) &= x_0 + \int_{z_0}^{z} [c(z)^2\sigma_x + \{1 - [v_x(z)\sigma_x + v_y(z)\sigma_y]\} \\ &\quad \times v_x(z)\,]\,[c(z)^2\sigma_z(z)]^{-1}\,dz \\ y(z) &= y_0 + \int_{z_0}^{z} [c(z)^2\sigma_y + \{1 - [v_x(z)\sigma_x + v_y(z)\sigma_y]\} \\ &\quad \times v_y(z)\,]\,[c(z)^2\sigma_z(z)]^{-1}\,dz \\ \sigma_x &= \cos\theta\,\{c(z_0) + v_x(z_0)\cos\theta + v_y(z_0)\sin\theta\cos\phi\}^{-1} \\ \sigma_y &= \sin\theta\cos\phi\,\{c(z_0) + v_x(z_0)\cos\theta + v_y(z_0)\sin\theta\cos\phi\}^{-1} \\ \sigma_z(z) &= [c(z)^{-2}\{1 - (v_x(z)\sigma_x + v_y(z)\sigma_y)\}^2 \\ &\quad - (\sigma_x^2 + \sigma_y^2)\,]^{0.5} \\ x_0 &= R\cos\theta \\ y_0 &= R\sin\theta\cos\phi \\ z_0 &= R\sin\theta\sin\phi \end{aligned} \tag{1}$$

where the positive x corresponds to flight direction, and the z axis is taken to be normal to the ground whose positive sense is downward. The polar and azimuthal angles θ and ϕ are measured from the positive x axis and the positive y axis to the ray, respectively. The term $c(z)$ is the local speed of sound at z, and $Vx(z)$ and $Vy(z)$ are the x and y component of wind velocity at z, respectively.